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"STUDY OF SOME FRACTIONAL TRANSFORMS AND IT'S APPLICATIONS"

Research Proposal submitted

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Mathematics

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I. Introduction:

Recently we found that Fractional Calculus is a new field for mathematical study that deals with the investigation and application of derivatives and integrals of non-integer orders.

In recent years, there has been a great deal of interest in Fractional differential equations. First there were almost no practical applications of fractional calculus and it was considered by many as an abstract area containing only mathematical manipulations of little or no use. Nearly, 30 years ago, the paradigm began to shift from pure mathematical formulations to applications in various fields. Important phenomenon in finance, electromagnetic, acoustics, viscoelasticity, electrochemistry and material science are well described by differential equations of fractional order. So far there have been several fundamental contribution on the fractional differential equations written by Oldham and Spanier (1974), Miller and Ross (1993), Podlubny (1999), Kilbas, Srivastava and Trujillo (2006).

Fractional differential equations appear in many branches of physics, engineering and applied mathematics. It has turned out that many phenomenon's in physics, engineering and other sciences can be described very successfully by models using mathematical tools from fractional calculus. For better understanding of a phenomenon described by a given nonlinear fractional partial differential equation, the solutions of differential equations of fractional order are much involved. Fractional derivatives provide more accurate models of real world problems than integer order derivative.

In our work we shall study basic properties of Fractional Calculus and prove some properties for calculus of non-integer order. Furthermore, we will study Fractional differential equations and try to solve linear and nonlinear Fractional differential equations by using different iterative and finite difference methods and compare the solutions. Also, we shall discuss the stability and convergence of the methods. For example, we will study Mittag-Leffler functions and their properties because analytical solutions of fractional differential equations directly occurs in the Mittag-Leffler function of one or two parameters. Also, we will study Fractional Laplace Transform and Fractional Fourier Transform along with their properties. We Solve initial Value Problems and

initial boundary value problems of fractional order and solve these by new iterative and finite difference methods.
We have the following definitions of Fractional Calculus for further developments

(a) Gamma Function : The Gamma function is defined as follows

$$\mathbb{\Gamma}(\mathrm{x}) = \int_0^\infty y^{x-1}e^{-y}\,dy, \qquad x > 0$$

where for convergence of the integer, x > 0

(b) Mittag- Leffler Function :

(i) Mittag - leffler function of one parameter is defined as follows

$$E_\alpha\,(z) = \sum_{k=0}^{\infty} \frac{z^k}{\mathbb{\Gamma}(\alpha k + 1)}\,, \quad (k\epsilon\mathbb{C}, \mathcal{R}(\alpha) > 0)$$

(ii) A two parameter function of the Mittag-Leffler type is defined by the series expansion as follows.

$$E_{\alpha,\beta}\,(z) = \sum_{k=0}^{\infty} \frac{z^k}{\mathbb{\Gamma}(\alpha k + \beta)}, (\alpha, \beta\,\epsilon\,\mathbb{C}, \mathcal{R}(\alpha) > 0, \mathcal{R}(\beta\,) > 0)$$

There are some relation of Mitag-Leffler functions to other functions

i) $E_0\,(z) = \frac{1}{1-z}\,, |z| < 1$

ii) $E_1\,(z) = e^z$

iii) $E_2(z) = \cosh\left(\sqrt{z}\right), \quad z \in \mathbb{C},$ etc

II. Purpose of carrying out Proposed research work:

Fractional Differential Equations arise as a mathematical modeling of systems and processes in the field of physics, chemistry, biology, economics, control system and image processing, bio-physics, aerodynamics, etc. The model problems are very difficult to handle and to obtain its analytical solutions. Recently, researchers develop some fractional order finite difference scheme

and iterative methods of fractional partial differential equation and obtain its solutions. We will develop some numerical methods for linear and nonlinear fractional partial differential equation to obtain their solutions.

III. Review of Literature and development in the Relevant subject :

Fractional Calculus is three centuries as old as the conventional calculus, that is, a system containing fractional derivatives and integrals. But remarkable work began only in the nineteenth century with many great mathematician like Euler, P. S. Laplace (1812), J.B.J. Fourier (1822), N. H. Abel (1823-1826), J. Liouville (1832-1873), B. Riemann (1847),, H. Holmgren (1865-67), A. K. Grunwald (1867-72), A. V. Letnikov (1868-1872), H. Laurant (1884), P.A. Nekrassov (1888), etc contributing to its development. However, recent attempt is on to have definition of Fractional Derivative as local operator specifically to fractal science theory. Perhaps, fractional calculus is the calculus of twenty first century. Heaviside (1922), Bus (1929), Goldmon (1949), Starkey (1954), Holbook (1966), Oldham and Spanier (1974), L. Debnath (1992), Milller and Ross (2005) and several others devoted additionally very reliable contributions to the subject.

IV. Research Gap :

Now a days, some of the properties of fractional calculus are not proved, we will try to prove such properties. Many Fractional Partial Differential equations do not have exact solutions so, researchers trying to obtain their numerical solutions. Therefore, we shall develop some numerical methods for solving linear and highly non linear Fractional Partial Differential Equations.

V. Importance of Study:

Partial Differential Equation plays a pivotal Role to describe nearly all systems that undergo change. Often, System described by Partial differential equations are so complex, or the systems that they describe are so large, that a purely analytical solution to the equation is not tractable. It is in these complex systems where computer simulations and numerical methods are useful. The techniques for solving partial differential equation based on numerical approximation were developed before programmable computer existed. Today, with the advent of high- speed super computers, computation has become a central tool of scientific progress.

Recently Fractional differential equations appears in many branches of physics, engineering and applied mathematics. It is found that many phenomenon in physics, engineering and other sciences can be described very successfully by models using mathematical tools from fractional calculus. For better understanding of a phenomena described by a given nonlinear fractional order are more involved Fractional derivatives provides more accurate models of real world problems than integer order derivative.

a. International status:-

Researchers and scientists have contributed in the development of theory of Fractional Calculus like Bus(1929), Goldmann (1949), Starking (1954), Holbrook (1966), Oldham and Spanier (1974), L. Debnath (1992), Miller and Ross (1993), R. K. Saxena (2002), Igor Podlubny (2003), S. Shaha Roy (2005) and several others devoted additionally very reliable contributions to the subject.

b. National status:-

Indian Mathematician Contributed in the development of Fractional Calculus like V. Srivastav, S. Das, K. C. Takale and D. B. Dhaigude, etc

c. Regional status : -

From Maharashtra many researchers have been actively involved in the study of Fractional Calculus like A. K. Pani, PSV Natraj, V. Dattar-Gejji, K. C. Takale and D. B. Dhaigude, etc

d. Scope of Study : -

Fractional Order derivatives and integrals provide a powerful instrument for the description of memory and hereditary properties of different diffusing substances. In the recent years, study of Fractional Partial Differential equation has become an active topic of research. But due to the complex geometries of most of the fractional differential equation they lack exact solution. Henceforth, the numerical techniques are widely preferred because of their accuracy and high computational efficiency. Few of them are Finite Difference Method, Adominas Decomposition Method, He's Variational Iteration Method, Homotopy Perturbation Method, Galerkin Method, Numerical Quadrature Method, Matrix Transform Method etc.

VI. Objectives of Proposed Research Work:

1. To study Basics properties of Fractional Calculus.
2. To study special functions such as Mitag-Leffler function.
3. To study Fractional Laplace Transform and Fractional Fourier Transforms
4. To study Fractional Differential Equations.
5. To study numerical methods of Fractional order.
6. To develop new numerical methods for solving linear and highly nonlinear Fractional Partial differential equations.
7. To publish our work in reputed Journals.

VII. Hypothesis-

The proposed work is useful for modeling real life phenomenon in the form of Fractional Differential Equations. In connection of this, in our proposed work we shall study some properties of Fractional Calculus. Furthermore, we will develop new iterative methods for modeled problems and try to solve by developed methods.

VIII. Limitation of work:-

1. Some of the properties of classical calculus do not hold in calculus of non-integer order

2. Many Fractional Differential Equations do not have the exact solutions.

IX. Research Methodology:

The following steps will be including in the research methodology:

1. To refer the relevant books on Fractional Calculus and Fractional Partial Differential Equations.
2. To study the Fractional Calculus.
3. To study Numerical Methods of integer order and fractional order.
4. To discuss the convergence of the Numerical methods.
5. To obtain the approximate solutions of Fractional Differential Equations.

X. Expected outcome:-

1. We will prove some properties of Fractional Calculus which are useful for further developments.
2. We will prove some properties of Fractional Laplace Transform and Fractional Fourier Transforms.

3. We shall find the numerical solutions of Fractional Partial Differential Equations and these will be represented graphically by mathematical Software.

XI. Proposed Work Plan/Formulation & Structure of Study:

(Year-Wise plan of work and targets to be achieved):

First year:

1. To refer the reference books on fractional calculus.
2. To prove some basic properties of Fractional Laplace Transform and Fractional Fourier Transform.

Second year:

1. To study numerical methods for differential equation.
2. To develop new numerical methods for linear Fractional Partial Differential Equations.
3. To compare solutions of integer order differential equations and Fractional order differential equations.
4. To discuss stability and convergence of the methods.

Third year:

1. To develop numerical methods for highly nonlinear fractional differential equations.
2. To discuss convergence of the numerical methods developed for highly nonlinear fractional partial differential equations.
3. We will represent the solution graphically by mathematical software's.
4. We will publish our work in reputed Journals.

XII. References:

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